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SEMI-EMPIRICAL PREDICTION OF BUBBLE DIAMETER IN GAS FLUIDIZED BEDS

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Abstract—Theoretical expressions for bubble diameter in both small and large particle fluidized beds are derived by the application of two phase theory and gas flow continuity. Comparison with experimental data suggests that the numerical and analytical solution of these expressions, combined with empirical bubble frequency relations, can provide an accurate prediction of bubble size and its parametric trends.

Several commonly employed empirical correlations of bubble diameter are shown to be derivable from a common theory, with differences among the correlations ascribed to variations in flow regime and bubble frequency.

INTRODUCTION

In gas fluidized beds, in which aggregative fluidization is generally encountered, the characteristics of the gas voids, which rise and grow within the emulsion, exert a profound influence on the fluid dynamics and chemical kinetics of the bed. Extensive investigations by Davidson *et al.* (Davidson & Harrison 1963), have shown that the dynamics of a single, isolated gas void in a fluidized medium are analogous to those of a gas bubble in a homogeneous liquid and that potential flow theory, as first developed by Davies & Taylor (1950), can be used to establish the velocity fields in the surrounding emulsion and in the rising bubble. However, the more complex characteristics of bubble swarms have not been thoroughly explored and the diameter, rise velocity and bubble concentration have generally been determined from empirical relations for similar beds, leaving major discrepancies unresolved.

Two-phase theory—particularly with Davidson's bubble model—forms the basis for much of the analysis of fluidized bed behavior (Davidson & Harrison 1963, Toomey & Johnstone 1952). In this representation, the emulsion, consisting of both solids and gas, is viewed as a single, composite phase while the gas voids are viewed as the second phase. An examination of the relevant physical relations suggests that the bubble diameter is the single most important bubble parameter and, indeed, many empirical correlations for bubble diameter are available in the literature and several of these are summarized in table 1. While all these relations provide adequate agreement with specific sets of measured values, they display striking differences in both general form and sensitivity to key variables. These discrepancies and the absence of a widely applicable predictive relation motivated much of a present effort to obtain a bubble diameter prediction based rigorously on two-phase theory and gas flow continuity.

In succeeding sections, theoretical expressions for the diameter of both slow and fast bubbles will be derived. Numerical and analytical solutions of the resulting relations will then be presented and compared with both data and several reported correlations.

THEORETICAL DEVELOPMENT

In the analysis of fluid dynamic phenomena in fluidized beds, it is often convenient to distinguish between "slow" bubbles, whose rise velocity is less than the interstitial gas velocity,

Table 1. Bubble size correlation (Darton *et al.* 1977) (dimensional constants given in SI units: h(m), U(m/sec), etc.)

Author	Correlation
Rowe	$D_f = (U - U_{mf})^{1/2}(h + h_0)^{3/4}/g^{1/4}$
Werther	$D_e = 0.00853[1 + 27.2(U - U_{mf})]^{1/3}(1 + 6.84h)^{1.21}$
Yacono	$D_f = 0.38h^{0.75}(U - U_{mf})^{0.41}$
Yasui and Johanson	$Y = 0.33\rho_s d_p[(U/U_{mf}) - 1]^{0.63}h$
Geldart	$D_f = \frac{1.43}{g^{0.2}}\left(\frac{(U - U_{mf})D^2}{4N_0}\right)^{0.4} + 2.05(U - U_{mf})^{0.94}h$
Darton *et al.*	$D_e = 0.54(U - U_{mf})^{0.4}(h + 4\sqrt{A_0})^{0.8}/g^{0.2}$

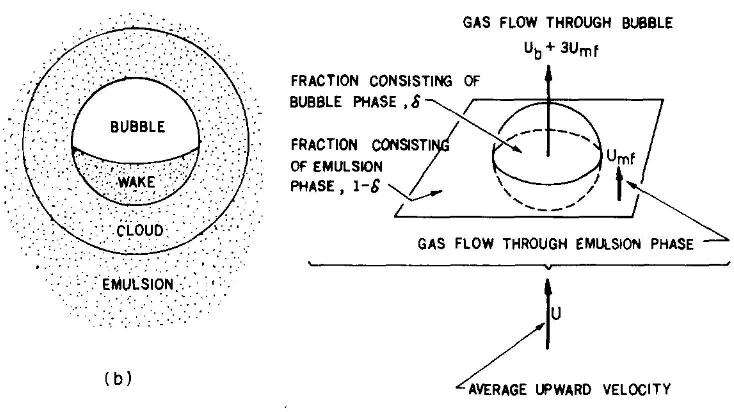
and "fast" bubbles, which rise at a velocity greater than the interstitial gas velocity (Davidson & Harrison 1963). Due to their relatively high minimum fluidization velocities, large-particle beds can be expected to operate both in the slow bubble and fast bubble regime, while the more common, fine-powder fluidized beds generally operate only in the fast bubble regime. In the analyses presented herein, distinct expressions will be derived for each of the two bubbling regimes, though it must be noted that "slow" and "fast" are relative terms and that a bubble of a given diameter could be slow in a bed of 1000-μm particles and fast in a bed of 50 μm.

Gas flow continuity

Gas transport through a fluidized bed operating in the slow bubble regime involves three distinct components (as depicted in figure 1a): interstitial gas flow in the emulsion phase, gas rising with the bubble in the recirculation and wake zones, and gas flow through the bubble void. Continuity considerations dictate that the flow rate of fluidizing gas equal the sum of these components. Combining the three components into a single relation, with the implicit assumption that the flow of gas around and through a single isolated bubble can be used to characterize the behavior of many bubbles rising simultaneously in an aggregatively fluidized bed—the superficial gas velocity is found to equal (Hughes 1978)

$$U = [1 - \delta(1 + \beta_x + \alpha)]\epsilon_{mf}U_e + \delta[(1 + \beta_x\epsilon_{mf} + \alpha\epsilon_{mf})u_b + 3(1 - \beta_n)U_{mf}] \quad [1]$$

where U is the superficial gas velocity; δ the fraction of bed volume occupied by bubbles; β_x the fraction of bubble volume occupied by the external recirculation zone; α the wake fraction (of bubble volume); ϵ_{mf} the value of bed voidage at minimum fluidization; U_e the velocity of the interstitial gas, u_b the bubble rise velocity; β_n the internal recirculation fraction (of bubble volume); and U_{mf} the superficial velocity at the incipience of fluidization. However, in the slow regime the wake fraction is suspected to be negligible (Geldart & Granfield 1972), and for low bubble rise velocities, the recirculation zones become negligible as well. Thus, for these



(a)

Figure 1. Bubble flow field: (a) slow bubble, (b) fast bubble.

conditions, [1] can be substantially simplified to the form originally suggested in Kunii & Levenspiel (1969)

$$U = (1-\delta)U_{mf} + \delta(u_b + 3U_{mf}). \qquad [2]$$

It is significant to note that a recent analysis of large particle data suggests that [2], in fact, is valid throughout the slow bubble regime and that the sum of the recirculation and throughflow terms is thus approximately constant and equal to $3U_{mf}\delta$ (Hughes 1978, Bar-Cohen *et al.* 1978).

Bubble gas flow in the fast bubble regime (represented schematically in figure 1b) involves gas recirculation in both the cloud and the wake as well as the volume of gas rising in the bubble. However, in contrast to slow bubble flow, it does not include flow of gas through the bubble void. Assuming once again that the isolated bubble flow field can be used to characterize the behavior of bubble swarms and combining bubble flow with the emulsion component, gas flow continuity for this regime can be expressed as

$$U = [1 - \delta(1+\beta+\alpha)]\epsilon_{mf}U_e + \delta[1 + (\alpha+\beta)\epsilon_{mf}]u_b \qquad [3]$$

where β is the fraction of bubble volume occupied by the cloud. In contrast to the slow bubble flow field, the fast bubble recirculation zone diminishes in size with *increasing* bubble rise velocity (Davidson & Harrison 1963), and consequently, for very fast bubbles, $u_{br} > 5U_{mf}/\epsilon_{mf}$, when gas flow in the wake and cloud becomes negligible, the superficial gas velocity can be closely approximated by (Kunii & Levenspiel 1969)

$$U = U_{mf}(1-\delta) + u_b\delta. \qquad [4]$$

The continuity equations for both fast and slow bubbles, [1]–[4], necessarily involve the absolute bubble rise velocity, u_b. Studies of isolated bubbles have established that the rise velocity of a single bubble in an otherwise undisturbed fluidized medium is given by (Davidson & Harrison 1963),

$$u_{br} = 0.711(gd_v)^{1/2} \qquad [5]$$

where u_{br} is the rise velocity of an isolated bubble, g the gravitational acceleration and d_v is the volumetric mean bubble diameter. The rise velocity of a bubble in a freely bubbling bed is subject to some uncertainty. The most commonly assumed relationship for the bubble rise velocity is (Davidson & Harrison 1963),

$$u_b = u_{br} + U - U_{mf}. \qquad [6]$$

Detailed measurements of u_b in bubble swarms by Godard & Richardson (1969), Werther (1975), Werther (1977), Botterill & Bloore (1963), Whitehead & Young (1967), appear to suggest that bubble velocities may differ considerably from values given by [6] but, in the absence of a more precise relation, this simple and commonly employed formulation will be used throughout the present discussion.

Following substitution of [6] into [2], the slow bubble continuity expression can be modified to yield a relation for bubble fraction, as

$$\delta_{Sb} = (U - U_{mf})/(0.711\sqrt{(gd_v)} + U + U_{mf}) \qquad [7]$$

with δ_{Sb} equal to the slow bubble fraction. Gas continuity for very fast bubbles, yields

$$\delta_{fb} = (U - U_{mf})/(0.711\sqrt{(gd_v)} + U - 2U_{mf}) \qquad [8]$$

where δ_{fb} is the fast bubble fraction.

Examination of [7] and [8] reveals that, even at specified values of U and U_{mf}, gas flow continuity can not provide an explicit value for bubble diameter but, rather, yields a relationship between the bubble fraction, δ, and bubble diameter, d_v. A second relation between bubble fraction and diameter is thus needed to theoretically determine bubble diameter at fixed values of superficial and minimum fluidization velcoities and it can be obtained by considering volumetric bubble flow at a given height within a fluidized bed.

Volumetric bubble flow

The volumetric bubble flow crossing a horizontal plane in a fluidized bed can be expressed as the product of bubble volume and the frequency, f_l, with which bubbles cross the reference plane or, alternatively, the product of bubble rise velocity and the area occupied by the bubbles. Thus,

$$(\pi d_v^3/6)f_l = u_b\delta A \quad [9]$$

where f_l is the frequency with which bubbles cross a given level in the bed and A is the crossectional area of the bed. For a random bubble distribution, a simpler measure of bubble frequency, namely the point frequency, defined as the rate at which bubbles strike a single immersed probe, can replace f_l in [9]. These two measures of bubble frequency are related through the ratio of bubble cross-sectional area to total bed area, i.e.

$$f_l/f_p = A/(\pi d_v^2/4) \quad [10]$$

with f_p equal to the point frequency.

Substituting the respective slow and fast bubble expressions for bubble fraction, obtained via continuity considerations, into [9] it is now possible to derive a relation for bubble diameter. Unfortunately, however, a theoretical bubble frequency relation is not presently available and empirical values have been reported in only a limited number of experimental studies. The few empirical frequency correlations appearing in the literature will, thus, be featured prominently in later sections.

Bubble diameter

In the slow bubble regime, the fraction of the bed occupied by bubbles is given by [7]. Cimbining [7] and [9], yields an algebraicly complex relation for slow bubble diameter, as

$$d_{vs} = [6A(U - U_{mf})/\pi f_l(1 + 2U_{mf}/(0.711\sqrt{(gd_v)} + U - U_{mf}))]^{1/3} \quad [11]$$

where d_{vs} is the diameter of a slow bubble. Similarly, the desired expression for bubble diameter in the fast bubble regime can be obtained by solving simultaneously [3] and [9], which for very fast bubbles yields (via [8])

$$d_{vf} = [6A(U - U_{mf})/\pi f_l(1 - U_{mf}/(0.711\sqrt{(gd_v)} + U - U_{mf}))]^{1/3} \quad [12]$$

where d_{vf} is the fast bubble diameter. When some measure of bubble frequency is available. [11] and [12] can be solved numerically by successive substitution of trial values of d_v, or, with the aid of judicious approximation, can be cast in a form amenable to analytic evaluation.

It is significant to note that, while several bubble diameter expressions are available in the literature, and a recent one due to Darton *et al.* (1977) achieves significant accuracy for small particle beds, none of these formulations attempts to satisfy gas flow continuity. In contrast, both [11] and [12] fully satisfy gas flow continuity as structured by the two-phase equations. The accuracy of the bubble diameters obtained by solving these equations will be examined in the next two sections.

NUMERICAL PREDICTION

Calculation procedure

To streamline the numerical solution of [11] and [12], it is desirable to establish *a priori* the range of possible bubble diameters and to adopt a rational search procedure for finding the value of d_v which correctly solves the algebraic equations.

Bubble diameter range. The relations used to derive [11] and [12] are based on the potential flow model for an isolated bubble and the two phase theory of fluidization. Although two-phase theory could allow the existence of vanishingly small bubbles, the presence of particle sized, inter-particle voids even in the minimally fluidized emulsions, establishes the particle diameter as a logical minimum value for bubble diameter.

In a fluidized medium the motion of an isolated bubble results in the creation of a recirculation zone (for slow bubbles) or a cloud zone (for fast bubbles) around each individual bubble. In the absence of a more rigorous solution, the isolated bubble, two-phase equations can be considered to apply as long as the spherical volume of the bubble, its wake and its cloud (or recirculation zone) does not overlap the adjacent bubble/wake/cloud volume. This approach was adopted in the present numerical calculations.

Since the present formulation is not meant to apply to slugging behavior, and as such behavior generally commences when the bubble diameter exceeds 1/3–1/2 the bed diameter (Stewart 1965), $d_v = D/2$ can serve an an upper limit on the range of bubble diameters to be explored in searching for the solution of [11] or [12].

Search procedure. With the d_v range defined, one of several widely used root-searching algorithms may be employed. The method of successive substitution is especially well-suited to the solution of [11] and [12] and was used in the present calculation (Hughes 1978). Once it is determined that a root exists within a sepcified range of d_v values, this method provides rapid convergence to the root when a monotonic dependence of the variable exists. Such a monotonic dependence of d_v on the superficial velocity is generally encountered in each of the possible d_v segments: the slow bubble range from d_p to bubble/wake/recirculation overlap and the fast bubble range from bubble/wake/cloud overlap to the slugging limit.

Comparison with data

Due to the dearth of slow bubble data in the literature and the inconsistency of bubble size measurements, complete validation of the two phase/frequency bubble growth model developed herein is most difficult. It is, nevertheless, possible to examine the accuracy of this bubble size prediction method by comparison with published experimental results.

Slow bubble regime. In the large-particle ($d_p \simeq 1760\,\mu$m) fluidized bed investigation by Cranfield & Geldart (1974) both bubble frequency and bubble diameter were measured by independent means and the bubble point frequency correlated according to

$$f_p = 16.7h^{-0.72} \pm 20 \text{ per cent} \qquad [13]$$

where h is the height above the distributor. The relative predictive capabilities of the present bubble growth model and Darton *et al.*'s (1977) fast-bubble formulation can thus be examined by comparison with Cranfield & Geldart's (1974) experimental d_v values. Such a comparison is shown in figure 2, where the values obtained by numerical solution of [11] are seen to be in excellent agreement with the data. Alternately, use of the method proposed by Darton *et al.* (1977) appears to result in a widening disparity as the measured bubble diameter increases, suggesting that it may be inadvisable to use fast-bubble formulations to predict bubble size in large particle beds.

In an earlier large particle ($d_p \simeq 1540\,\mu$m) fluidization study, performed by McGrath & Streatfield (1971), the diameter and frequency of the bubbles erupting from a vigorously bubbling bed were measured, although with less accuracy than in Cranfield & Geldart (1974). Examina-

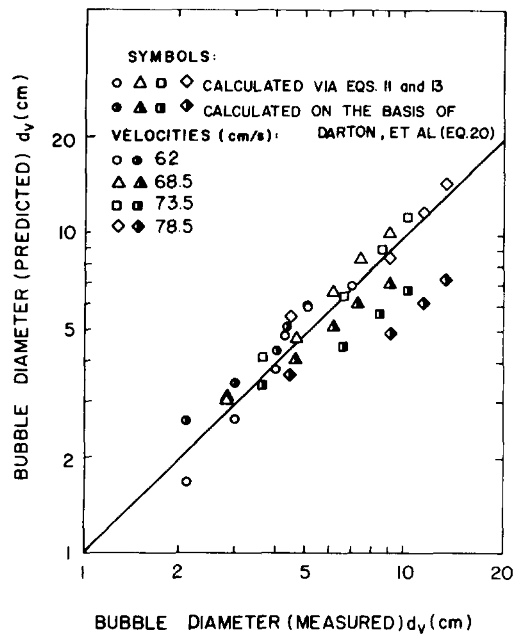


Figure 2. Bubble diameter predictions, data of Cranfield & Geldart (1974).

tion of figure 3 reveals that the solution of [11] using the measured McGrath and Streatfield frequency, yields moderately good agreement between predicted and measured bubble diameters, the latter taken to equal 2/3 of the observed eruption diameters. The modest disparity noted could perhaps be accounted for by experimental inaccuracies. The other two predictions shown in the figure, obtained by solving [11] with the Cranfield & Geldart (1974) frequency correlation and the Darton *et al.* (1977) model, respectively, are seen to offer

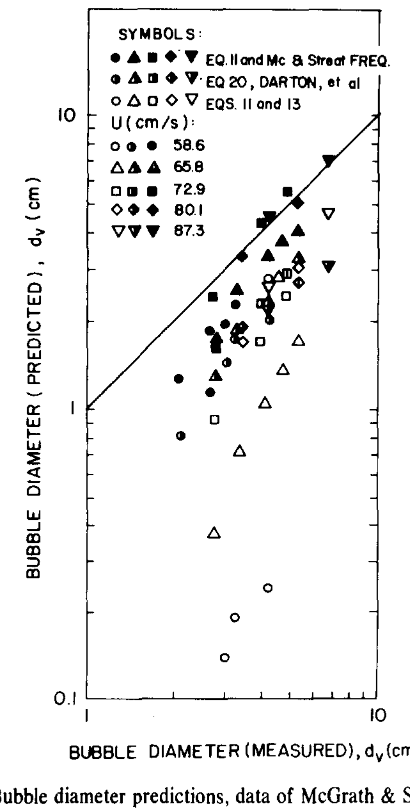


Figure 3. Bubble diameter predictions, data of McGrath & Streatfield (1971).

significantly poorer agreement with the data, especially for small bubble diameters. These results can be traced to differences in bubble frequency between the two large particle studies and suggest again that fast-bubble correlation may not be applicable to beds operating in the slow bubble regime.

Fast bubble regime. In a study of small particle ($d_p \simeq 103\ \mu$m) fluidization, Werther (1976) measured both volumetric mean bubble diameters and bubble frequency as functions of fluidizing velocity and bed height. The measured bubble diameters place this study well within the fast bubble regime and Werther's empirical correlation of bubble frequency, typically offering ± 20 per cent agreement with data,

$$f_l/A = (0.57 + 0.39h)^{-3} \quad [14]$$

must, therefore, be viewed as primarily appropriate to this regime of bed behavior. The excellent agreement between predicted bubble diameter, obtained by solution of [12] with the frequency of [14], and the measured values is shown in figure 4. Significantly, the Darton *et al.* (1977) correlation, developed in part on the basis of Werther's (1976) data, is in this regime seen to offer equally good agreement. Values predicted on the basis of the Cranfield & Geldart (1974) frequency, [13] are, on the other hand, seen to diverge widely from the measured fast bubble diameters.

Rowe & Everett (1972), in their study of small particle fluidization (d_p varying from 135 to 323 μm), used X-rays of portions of the bed to determine both the number of bubbles and the mean bubble diameter in the control volume. Two examples of a comparison between their measured values and various bubble diameter prediction techniques are shown in figures 5 and 6. As can be seen in these figures, agreement between solutions of [12] (using reported bubble frequency) and the data is good but not as close as the predictions of the Darton *et al.* (1977) model. Bubble diameter values derived via [12] and [13] (the Cranfield and Geldart frequency) offer acceptable accuracy only at the low end of the bubble size spectrum, closest to the slow bubble regime.

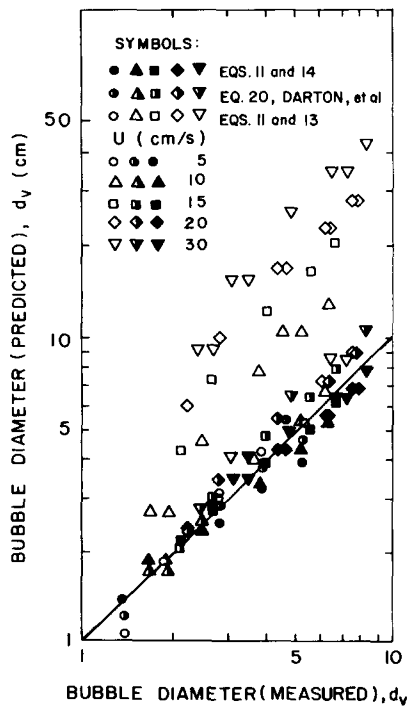


Figure 4. Bubble diameter predictions, data of Werther (1976).

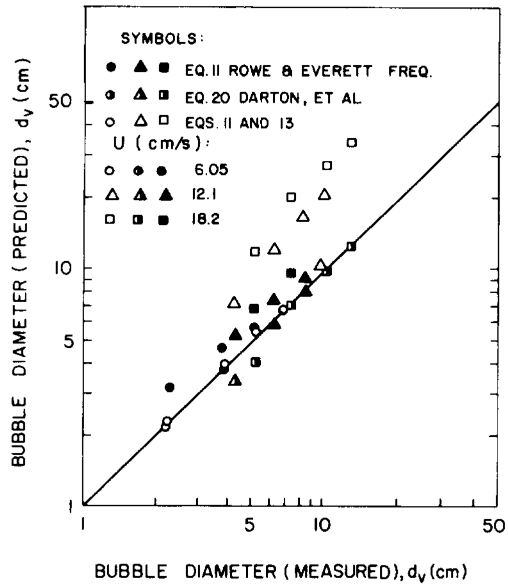


Figure 5. Bubble diameter predictions, data of Rowe & Everett (1972)—quartz.

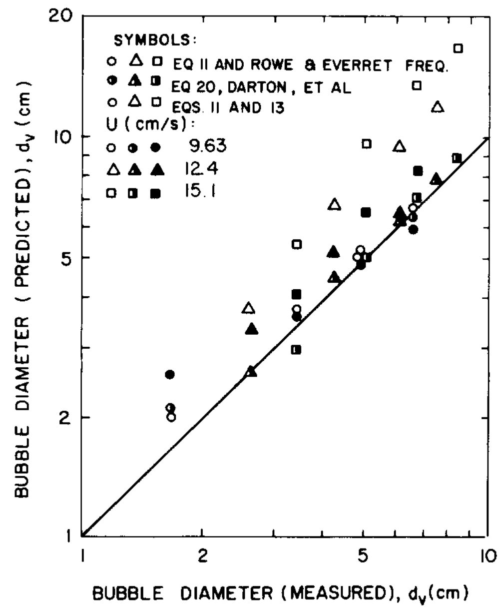


Figure 6. Bubble diameter predictions, data of Rowe & Everett (1972)—glass powder.

DEVELOPMENT OF SEMI-EMPIRICAL RELATIONS

Examination of available empirical and semi-empirical correlations for bubble diameter, d_v, in fluidized beds (Rowe 1976, Werther 1976, Yasui & Johanson 1958, Kato & Wen 1969, Geldart 1972, Cranfield & Geldart 1974, Darton *et al.* 1977, Park *et al.* 1969) reveals striking differences in both the form and specific dependence of d_v on key variables, as can be seen in table 1 presented earlier in the discussion. Yet, much of the data used to obtain three of the more prominent correlations—Cranfield & Geldart's (1974) for large particles and both Werther's (1976) and Darton *et al.* (1977) for small particles— has been shown above to agree with two phase theory. It would thus appear possible to use the two phase/frequency approach to derive approximate expressions for bubble diameter that are compatible with the present understanding of fluidization phenomena and offer a rational basis for the correlation of future experimental results. In keeping with the slow/fast bubble classification of convenience employed in previous sections, the derivation and exploration of such expressions will be done separately for large and small particle systems, respectively.

Large particle systems

In the most comprehensive large particle study to date (Cranfield & Geldart 1974) the frequency with which bubbles struck a point probe and the bubble diameter were measured independently of each other. The diameter data, shown in figure 2 to be in excellent agreement with two-phase theory, were correlated by the investigators in the form

$$d_v = 0.0326(U - U_{mb})^{1.11}h^{0.81} \pm 10 \text{ per cent} \qquad [15]$$

where U_{mb} is the superficial velocity at bubbling incipience.

Introducing [10] and [11] and substituting the empirical point frequency relation given by Cranfield & Geldart (1974) ([13]), the theoretical slow bubble d_v expression can be modified to yield

$$d_{vs} = 0.0898(1 + 2U_{mf}/u_b)^{-1}(U - U_{mf})h^{0.72}. \qquad [16]$$

Alternatively, it is possible to recast [16] in the form

$$d_v = C_1(U - U_{mf})h^{0.72} \qquad [17]$$

where C_1 is the average value of $(0.0898)(1 + 2U_{mf}/u_b)^{-1}$ for the slow bubble regime. Depending on the specified parametric range encountered in a given apparatus and/or set of experiments, the value of C_1 can be expected to take on somewhat different values. However, a reasonable estimate for the average (slow bubble) value of C_1 can be obtained by setting u_{br} equal to one-half the slow/fast transition velocity, i.e. $u_{br} = U_{mf}/2\epsilon_{mf}$, and U/U_{mf} equal to 1.5 to yield $C_1 \simeq 0.04$. With this value of C_1, [17] is seen to be quite similar to the empirical bubble diameter correlation of Cranfield & Geldart (1974), with U_{mf} replacing the minimum bubbling velocity U_{mb}. The values of d_v predicted by [17] with $C_1 = 0.04$ and the Cranfield and Geldart empirical correlation are both compared with Cranfield & Geldart (1974) data in figure 7. As might be expected from the use of an average C_1 value, bubble diameters predicted by the analytical, two-phase relation ([17]) scatter on both sides of the data points, but most

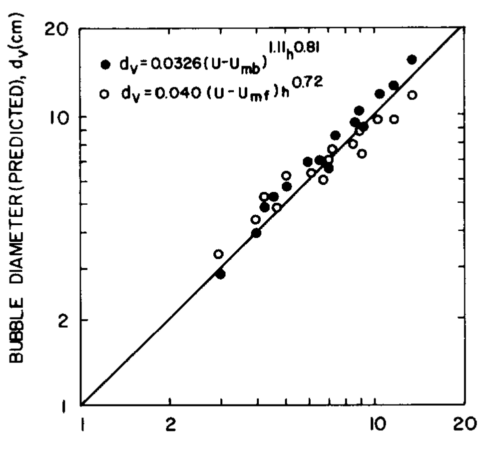


Figure 7. Comparison of bubble diameter predictions with data of Cranfield & Geldart (1974).

importantly offer comparable overall agreement with data to that obtained by use of the empirical correlation ([15]).

Small particle systems

In the small particle fluidization study by Werther (1976) bubble size, velocity and point frequency were measured and used to determine the level frequency and d_v. The values for level frequency were found to vary inversely as the height to the third power (as shown by [14]) and the bubble diameter correlated to generally within ±10 per cent, by

$$d_v = 0.853[1 + 0.272(U - U_{mf})]^{1/3}[1 + 0.0684h]^{1.21}. \qquad [18]$$

Returning to [12] the fast-bubble analytical expression for d_v, it is noticed that for values of u_b much greater than U_{mf} (as is generally the case in this regime) the denominator approaches πf_l, i.e. $1 - U_{mf}/u_b \to 1$. Consequently, with only a modest loss of accuracy, [12] can be greatly simplified and following substitution for f_l from [14] shown to yield

$$d_{vf} = 0.71(U - U_{mf})^{1/3}(1 + 0.0684h). \qquad [19]$$

Examination of [19] reveals that it is of the same general form as the empirical correlation, though, as before, differences do exist in the precise values of the coefficients and the power dependence of d_v on bed height. Interestingly, while Werther's (1976) empirical correlation, [18], and [19] show approximately the same dependence on excess velocity, $U - U_{mf}$, for large values of this parameter; the derived semi-empirical expression predicts a zero bubble diameter at the minimum fluidization condition, while the empirical correlation yields a finite bubble diameter even for U less than U_{mf}.

Comparison of [19] and [18] with the data obtained by Werther (1976) reveals, as can be seen in figure 8, that the theoretical relation offers acceptable though somewhat poorer agreement with the data than achieved with the correlation itself. Part of the disagreement of [19] with the data may be due to the simplifying approximations made in the derivation.

In the previously referred to Darton *et al.* (1977) study, the coalescence of neighboring streams was assumed to be the primary mechanism for bubble growth in fluidized media and coalescence was postulated to occur after each bubble had risen a distance λD_c in the medium, where λ is an empirical constant and D_c the diameter of the "catchment" area of each bubble

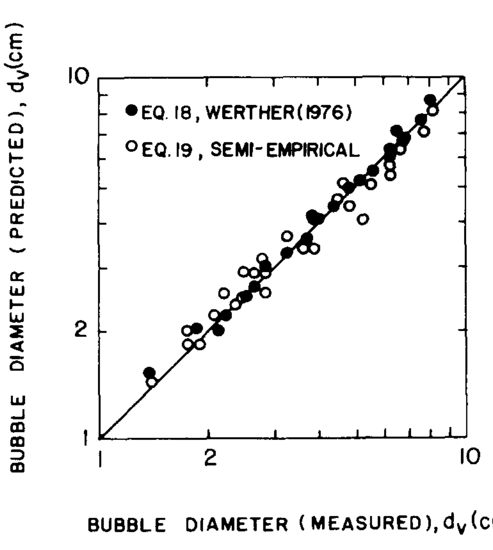


Figure 8. Comparison of bubble diameter predictions with data of Werther (1976).

and related to the distance separating adjacent bubbles. Based on this postulate, two phase theory and assumed bubble volume conservation, as well as ancillary assumptions, the bubble diameter was found to equal (Darton *et al.* 1977)

$$d_v = 0.54(U - U_{mf})^{0.4}(h + 4\sqrt{A_0})^{0.8}/g^{0.2} \quad [20]$$

where A_0 is the catchment area at the distributor. The coefficient shown resulted from setting $\lambda = 1.17$, obtained by examination of published measurements of bubble size, particularly Werther (1976) and Rowe & Everett (1972). While [20] offers general agreement with the data it overpredicts the d_v values of Werther (1976) by as much as 30 per cent and those of Rowe & Everett (1972) by approx. 10 per cent (Darton *et al.* 1977).

The basic Darton *et al.* (1977) postulate concerning the proportionality of bubble coalescence height to initial distance of separation, provides in effect, an alternative bubble frequency formulation which can be used, in the manner described in the earlier sections of this paper, to derive a bubble diameter relation consistent with both two phase theory and gas flow continuity and free of the unproven assumption of bubble volume conservation. To aid in this process, it is convenient to note that the bubble fraction can be related to the number of bubbles contained in a cross-sectional slice of height kd_v in the bed, according to

$$\delta = N''\left(\frac{\pi}{6}d_v^3\right)\bigg/kd_v. \quad [21]$$

In [21], N'' represents the number of bubbles per unit area, at a given height in the bed, while kd_v equals the height of the bubble and its wake or cloud. Although k is not well defined, a value of 1.1 would be consistent with a fast bubble wake fraction of 1/3, as suggested by Rowe & Partridge (1965).

The Darton *et al.* (1977) postulate on successive bubble coalescence at increments of λD_c can be used to relate the number of bubbles (per unit area) at height h in the bed to the number at height h_0. Following Hughes (1978)

$$N''(h) = N''_0[1 + (\sqrt{2} - 1)(h - h_0)/\lambda D_{c0}]^{-2} \quad [22]$$

where D_{c0} is the catchment area at the distributor and equal to $\sqrt{(4A_0/\pi)}$ or $\sqrt{(4/\pi N''_0)}$. Comparison of this relation with the data of Werther (1976) and Rowe & Everett (1972) shows λ to vary from approx. 0.5 to 2.45 and to average 1.7 for the two studies cited (Hughes 1978).

Combining [21] and [22], with $k = 1.1$ and $\lambda = 1.7$, yields an expression for fast bubble fraction which can be equated with δ_{fb} obtained from [8]. This latter operation eliminates δ and yields

$$d_v = [(3)(1.1)/2]^{0.5}[U - U_{mf}]^{0.5}[(\sqrt{2} - 1)(h - h_0)/1.7 + D_{c0}]/[0.711\sqrt{(gd_v)} + U - 2U_{mf}]^{0.5}. \quad [23]$$

In the fast bubble regime, and especially for the range of large bubble sizes encountered by Werther (1976) and Rowe & Everett (1972), the denominator of [23] can be shown to approach $(0.711\sqrt{(gd_v)})^{0.5}$. With this approximation and some algebraic manipulation, [23] yields an explicit expression for bubble diameter

$$d_v = 0.45(U - U_{mf})^{0.4}[h + 4.63\sqrt{A_0})^{0.8}/g^{0.2}. \quad [24]$$

This expression is very similar to the Darton *et al.* (1977) relation and the lower coefficient (0.45 vs 0.54) in [24] can be expected to offer better agreement with Werther (1976) and Rowe & Everett (1972) data in the upper reaches of the respective beds than achieved by [20].

CLOSURE

The preceding development of a model for bubble growth in both large and small particle fluidized media has attempted to provide a rigorous basis for the correlation of data and prediction of bubble diameter in fluidized beds. The good to excellent agreement obtained in comparing the numerical solutions and semi-empirical correlations with the limited data available, appears to validate the use of gas flow continuity—as structured by two phase theory and coupled with some measure of bubble frequency—to achieve this goal.

Unfortunately, however, the utility of the two-phase/continuity/frequency approach is limited by the absence of a theoretical bubble frequency relation and dearth of empirical values. More precise prediction of bubble diameter in gas fluidized beds must thus await the development of a detailed mechanistic model of bubble formation, coalescence and collapse which will yield a theoretical relation for bubble frequency. Alternately, since point frequency can be measured relatively easily, the foregoing methodology can be used in conjunction with measured point frequencies to predict bubble characteristics in large, industrial fluidized beds.

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